Use of a vertical through-flow to stratify an initially isothermal contained fluid

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A study is made of the development of stratification in an initially homogeneous fluid contained in a vertically mounted thermal storage tank when a vertically upward through-flow is forced to pass through the system. The one-dimensional analytical model of Rahm and Walin is briefly recounted. The through-flow facilitates convective circulation inside the tank, thereby enabling the system to approach the steady state in a time considerably shorter than the diffusive time scale. The verification studies of the model are reviewed. By numerically integrating the model equation, the evolution of the temperature field in an initially isothermal fluid is examined. Physical descriptions are provided on how the stratifying process is affected by the effects of the through-flow magnitude, of the thermal conductance of the tank sidewall, and of the thermal diffusivity of the fluid.

Keywords: thermal energy storage tank; stratification control; through-flux; thermal convective process

Introduction

Prediction and control of stratification in a contained fluid is an important topic for a host of thermal engineering applications and geophysical fluid modeling experiments. Specifically, the stratification control in the thermal energy storage tank can lead to a significant improvement in the overall efficiency of a solar thermal system (see, e.g., Refs. 1, 2). One well-known example is the thermocline thermal energy storage tank.[2] This system consists of a single tank which stores sensible energy in a fluid. This storage tank contains both warm fluid from the solar collector and chilled fluid from the thermal load by maintaining thermal stratification. Another example of the use of the storage tank can be found in the operation of the heating and air conditioning systems for large buildings. The demand for chilled water is high during the daytime, and hot water is needed to heat the building at night. A considerable energy savings can be expected if a thermal energy storage tank is effectively utilized. In all these applications, it is essential to control thermal stratification in the tank. A variety of schemes have been devised with a view toward controlling and/or enhancing fluid stratification in an enclosed vessel. One currently popular method calls for the use of fluid fluxes in and out of the system boundaries.[1,3,4] Among others, Gari and Loehrke[3] designed an inlet manifold which introduces a controlled buoyant fluid jet into a thermal storage tank; Baines *et al.*[4] conducted experiments to describe the development of stratification caused by horizontal inflow in a two-dimensional rectangular tank.

The above-mentioned studies demonstrated that the use of fluid fluxes is indeed an effective way to enhance fluid stratification. It has also been found that the temperature distribution in the main body of fluid is essentially one-dimensional, varying principally in the vertical direction. Taking advantage of the one-dimensional nature of the interior temperature field, Rahm and Walin[5,6] proposed a simple theoretical model to predict the fluid stratification when the overall Rayleigh number for the fluid system is large. In this model, the nonhorizontal boundaries of the container are allowed to have a finite thermal conductance. The crucial part of the model is that, in order to facilitate the development of fluid stratification, a small upward vertical mass flux, M_0, is forced to pass through the porous horizontal boundaries. The temperature and the magnitude of the flux at the horizontal boundaries are controlled. The purpose is to utilize convection as the primary dynamical mechanism; accordingly, the system adjustment time to reach the steady state is substantially reduced below the diffusive time scale. Based on elaborate scaling arguments and boundary layer analyses, Rahm and Walin,[5,6] derived a differential equation for the lowest-order interior temperature.

The temperature profiles predicted by this model have been verified for a vertically mounted circular cylindrical tank whose axis is parallel to gravity g. Gross[2] reported favorable comparisons between the model predictions and the large-scale static tests in a completely closed thermocline storage tank ($M_0 = 0$). The model results for steady-state temperature profiles showed close agreement with the preliminary laboratory measurements using moderate values of M_0.[5,6] Recently, the model was brought under further scrutiny by comparing its temperature predictions against the numerical solutions to the Navier-Stokes equations for a Boussinesq fluid having a preexisting stratification.[7] It was shown that both the transient and steady-state temperature fields predicted by the model were consistent with the numerical solutions. The numerical studies showed that, with a proper choice of the through-flow M_0, a virtually motionless interior core with a near-linear stratification can be attained.

The above-mentioned verification studies have established that this simple analytical model provides an adequate description of the interior temperature distribution; it captures the qualitative essentials of the thermal convective adjustment process of a contained fluid. In practical terms, the model offers

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a significant new idea leading to an efficient laboratory technique to produce stratified fluid systems.

In this paper, we shall apply the validated model equation of Refs. 5 and 6 to a problem of considerable practical relevance; we shall describe the stratification buildup in an initially isothermal fluid in a cylindrical tank when a vertical through-flow M_0 is impulsively imposed. Knowledge of the transient phase is needed for the design and operation of thermal energy storage tanks.[1] The problem is also important to the setup of certain geophysical fluid experiments.[6–10] Discussion will be focused on how the evolution of the temperature field in the main body of fluid is influenced by the various dynamical effects, namely, the effects of the magnitude of the through-flow, of the vertical sidewall thermal conductance, and of the thermal diffusivity of the fluid.

The model equations

In this section, the theoretical model of Rahm and Walin[5,6,8,9] will be briefly recounted. Consider a vertically mounted cylinder of radius a and height h, filled with a Boussinesq fluid. The relevant fluid properties are α, coefficient of volumetric expansion; ν, kinematic viscosity; and κ, thermal diffusivity. These properties are taken to be constants. The radial and vertical coordinates are denoted by (r, z). At the initial state, the fluid is at rest with a prescribed temperature profile $T_i(z)$. At $t=0$, an upward vertical through-flux M_0 is forced to pass uniformly through the porous horizontal endcap disks and this condition is maintained so thereafter. The endcap disks are thermal conductors. The temperatures are set to be T_b at the bottom inlet disk at $z=0$ and T_t at the top outlet disk at $z=h$. Note that $\Delta T \equiv T_t - T_b > 0$ to insure a gravitationally stable configuration. The crux of the model is that the vertical sidewall is allowed to have a finite thermal conductance. Accordingly, a Newtonian heat flux condition is applied[5,6,8]:

$$\frac{\partial T}{\partial r} = f(T_e - T) \qquad \text{at } r=a \tag{1}$$

where T_e is the temperature of the environment. Here, it is assumed that T_e is intermediate between T_b and T_t. The thermal conductance of the sidewall is represented by f. Physically, $f = k_w/kd$, where k_w and k denote the thermal conductivities of the sidewall material and of the fluid, and d is the thickness of the sidewall.[6]

The model is formulated under the following conditions: (a) the system Rayleigh number, $\mathrm{Ra} \equiv \alpha g \Delta T h^3/\nu\kappa$, is sufficiently large to render a boundary-layer-type flow; (b) the thermal forcing at the vertical sidewall is limited; (c) the fluid system is strongly stratified. These imply that

$$\frac{\nu}{Nh^2}, \quad \frac{\kappa}{Nh^2} \ll 1, \quad \mathrm{Pr} \sim O(1) \tag{2a}$$

$$fh \ll \left(\frac{\kappa}{Nh^2}\right)^{-1/2} \tag{2b}$$

$$T^{\mathrm{I}} = T^{\mathrm{I}}(z, t), \quad w^{\mathrm{I}} = w^{\mathrm{I}}(z, t) \tag{2c}$$

where $N \equiv (\alpha g \Delta T/h)^{1/2}$ is the buoyancy frequency, w the vertical velocity, and the superscript I pertains to the interior core region.

The one-dimensionality assumption (2c) affords substantial simplifications in the mathematical manipulations. Considering the fluid transport carried by the sidewall boundary layer, M_b, and the global mass continuity throughout the container, i.e., $M_b + w^{\mathrm{I}}\pi a^2 = M_0$, Walin[8] arrived at a differential equation for the lowest-order interior temperature T^{I} (see Equation 4.14 of Ref. 8). Introducing the dimensionless quantities

$$z' = \frac{z}{h}, \quad t' = \frac{t}{t_h}, \quad S = \frac{T - T_b}{T_t - T_b}$$

$$A_r \equiv \frac{M_0 t_h}{\pi a^2 h}, \quad B_r \equiv \frac{2\kappa t_h f}{a}, \quad C_r \equiv \frac{\kappa t_h}{a^2} \tag{3}$$

we obtain the equation for the interior temperature S (dropping primes from z' and t'):

$$\frac{\partial S}{\partial t} + A_r \frac{\partial S}{\partial z} + B_r(S - S_e) = C_r \frac{\partial^2 S}{\partial z^2} \tag{4}$$

In Equation 4, S_e denotes the nondimensional counterpart to T_e.

In Equation 3, t_h is the characteristic time scale of the adjustment process, which is yet to be determined. Physically, the coefficients A_r, B_r, and C_r in Equation 3 can be interpreted, respectively, as the ratio of the characteristic time to the flush time due to the imposed through-flow M_0, to the flush time due to the buoyancy layer transport, and to the diffusive time.[9] Here, we are concerned with the advection dominant regime. Therefore, the time scale for the overall adjustment process is mainly governed by the flush time due to the boundary layer transport.[5,6,8,9] This consideration leads to $t_h \sim a/2\kappa f$ so that $B_r \sim O(1)$, $C_r \ll 1$. It follows that, since M_0 and M_b are comparable in magnitude[8,9], we have $A_r \sim O(1)$, pointing to $M_0 \gg \kappa\pi a^2/h$.

The model validations

The interior temperatures predicted by Equation 4 were verified previously by laboratory measurements (see Figure 13 of Ref. 2 and Figure 4 of Ref. 6). Also, the numerical solutions of the full Navier-Stokes equations were supportive of the major contentions of the theoretical model equation 4 (see Ref. 7). Figure 1 displays one such representative comparison. Figure 2 presents the numerically constructed plots of the stream function in the meridional plane. The details of the flow field are not readily available from the theoretical model equation.

Rahm[9] offered an informative physical description of the qualitative behavior of the fluid system. Initially, the cold fluid that is pumped into the container through the bottom inlet disk passes through the interior region of the container. The interior

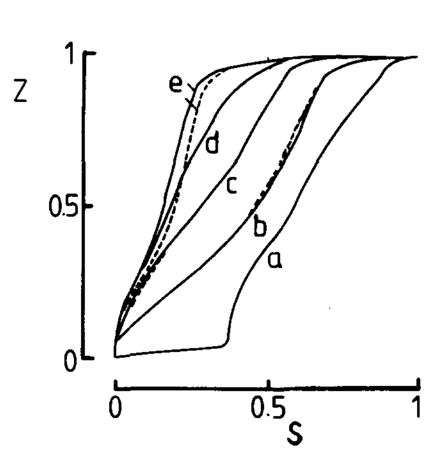

Figure 1 Representative comparison between the model predictions (-----) and the numerical solutions (———). The preexisting, initial temperature profile is $S_i(z) = 0.3 + 0.6z$. $A_r = 1.0$, $B_r = 1.0$, $C_r = 0.02$. Times t are (a) 0.01; (b) 0.24; (c) 0.49; (d) 0.76; (e) 1.41

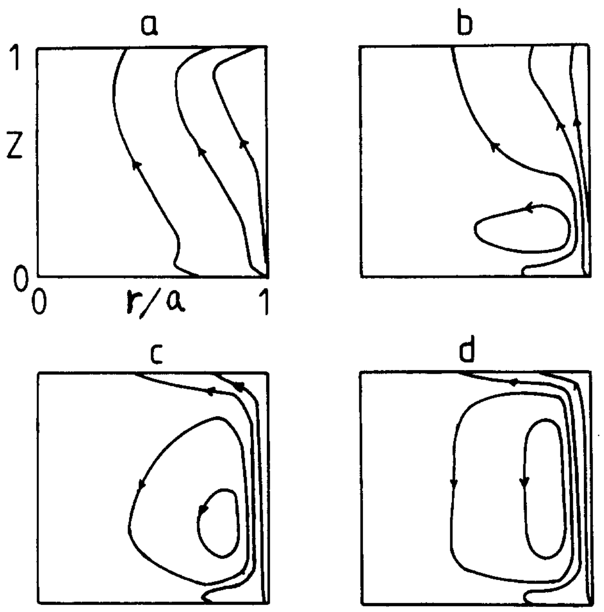

Figure 2 Plots of the stream function ψ in the meridional plane based on the numerical solutions shown in Figure 1. The values of ψ are in units of M_0/π, and the contour increment $\Delta\psi = 0.32$: (a) $t=0.20$, $\psi_{max}=0$, $\psi_{min}=-1.04$; (b) $t=0.54$, $\psi_{max}=0.40$, $\psi_{min}=-0.84$; (c) $t=1.07$, $\psi_{max}=0.60$, $\psi_{min}=-0.83$; (d) $t=1.75$, $\psi_{max}=0.62$, $\psi_{min}=-0.83$

fluid that initially occupied the container is lifted toward the top outlet disk. The dividing line between the original fluid and the fluid of the through-flow now forms a vertically moving temperature front in the interior. The appearance of a moving temperature front was also observed in the thermal adjustment process in a closed vessel.[6,10]. In the shrinking region between this front and the outlet disk, the upward motion of the original fluid is spread throughout the interior. In the expanding region between the bottom inlet disk and the front, the interior velocity is weak and is directed downward.[6] In this region, the imposed through-flow M_0 takes route via the sidewall boundary layer. The flux enters the interior at the level of the front. In time $t \sim O(1)$, the front reaches the top outlet. In the steady state, the vertical flux of the imposed through-flow is carried almost completely by the sidewall boundary layer transport, i.e., $M_b \cong M_0$, at every level. This condition determines the interior temperature field (see Equation 3.6 of Ref. 6). We recall that in Equation 4, $C_r \ll 1$, indicating that the characteristic time for the adjustment process is much smaller than the diffusive time h^2/κ.

The physical pictures described above are consistent with the flow and temperature fields obtained by the numerical solutions, as exemplified in Figures 1 and 2; the essential qualitative correctness of the model has now been verified.

Results of the model equations

We now adopt the validated model Equation 4 to depict the development of stratification in an initially isothermal fluid in a cylindrical tank. Physically, the specific flow situation can be modeled in the following way. Initially, the fluid inside the tank is motionless and at a uniform (dimensionless) temperature, $S_i(z)=0.5$; initially, the fluid is in thermal equilibrium with the environment, $S_e=0.5$. At $t=0$, the temperatures of the endcap disks are abruptly set respectively to $S_b=0$ and $S_t=1$; at the same time, an upward vertical through-flow, M_0, is forced to pass through the porous horizontal disks. These boundary conditions will be held constant thereafter.

Equation 4 is readily integrated by employing a Crank-Nicolson scheme. The spatial resolution was typically $\Delta z = 0.025$, and the time step was $\Delta t = 0.005$. The integration of Equation 4 continued to the approximate steady state, which is defined arbitrarily to be the state when the maximum value of $\partial S/\partial t$ falls below 10^{-4}. The results are presented in Figures 3–10. The curve denoting the largest time in the plots corresponds to the above-defined steady state.

Figure 3 shows the temperature evolution for the case of $A_r=1.0$, $B_r=1.0$, $C_r=0.02$; this will serve as a benchmark case. The existence of the temperature front is apparent in the figure. The front is seen to traverse the entire vertical extent of the tank within $t \sim O(1)$. Ahead (upward) of this front, the interior fluid remains isothermal ($S=0.5$). Behind (downward of) the front, the fluid is being stratified. At early times, the temperature gradient is large in the stratified region. As time progresses, the stratified region widens, and the temperature gradient in this region decreases accordingly. Since the stratified region encompasses the bottom inlet disk, no thermal boundary layer is seen near the inlet disk.[5,6] A thin thermal boundary layer forms near the downstream (outlet) disk to adjust the interior temperature to T_t, which is externally impressed on the top

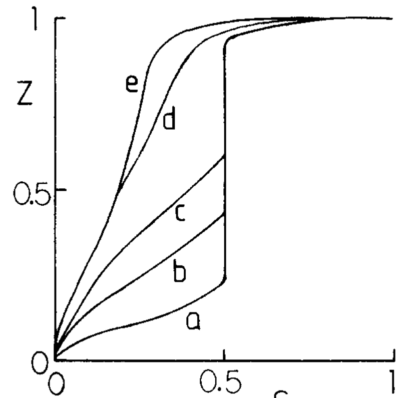

Figure 3 The model predictions of the interior temperatures for an initially isothermal fluid. $A_r=1.0$, $B_r=1.0$, $C_r=0.02$. Times t are (a) 0.10; (b) 0.25; (c) 0.40; (d) 0.80; (e) 1.45

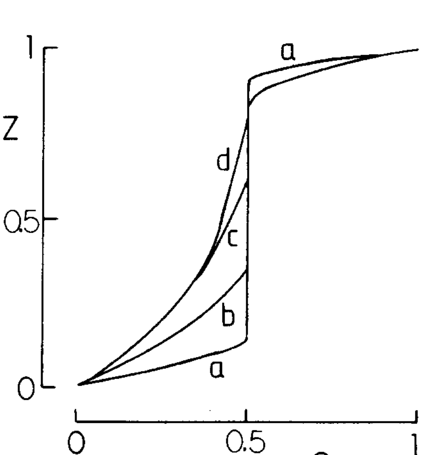

Figure 4 Same as in Figure 3, except $A_r=0.2$, $B_r=1.0$, $C_r=0.02$. Times t are (a) 0.10; (b) 0.50; (c) 1.50; (d) 2.73

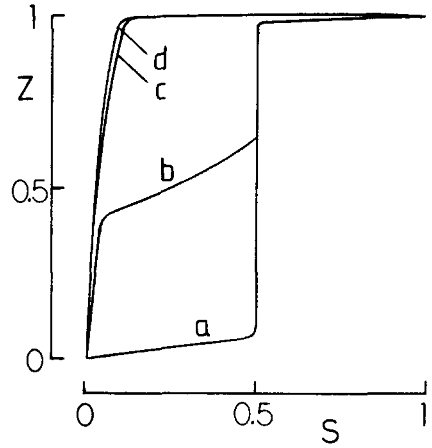

Figure 5 Same as in Figure 3, except $A_r=5.0$, $B_r=1.0$, $C_r=0.02$. Times t are (a) 0.01; (b) 0.10; (c) 0.30; (d) 0.70

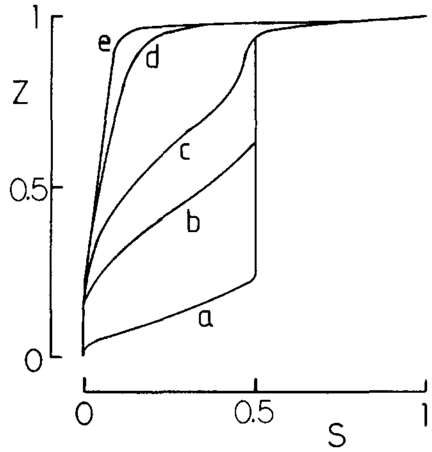

Figure 6 Same as in Figure 3, except $A_r=1.0$, $B_r=0.2$, $C_r=0.02$. Times t are (a) 0.10; (b) 0.40; (c) 0.60; (d) 1.0; (e) 1.56

endcap disk. The dimensional time needed to attain the approximate steady state is scaled by the characteristic time t_h. Recalling that $C_r\ [\equiv t_h\kappa/a^2]=0.02\ll 1$, we recognize that the time to reach the steady state is much smaller than the diffusive time scale a^2/κ. This is in support of the idea that the use of an upward mass flux can be an efficient way to produce a stratified fluid system.

Figures 4 and 5 illustrate the effect of the magnitude of the imposed through-flow, M_0, which is manifested in A_r. As is discernible in Figure 4, when M_0 is small, the convective activities weaken, and, consequently, the relative importance of conduction increases. It is also seen that the time to reach the steady state increases as M_0 decreases. The thermal boundary layer near the top outlet disk becomes thicker. As M_0 decreases further, in the steady-state limit, the temperature profile in the main body of fluid tends to the linear profile obtainable when conduction is the dominant mechanism. On the other hand, the physical implications of a larger M_0 are that the through-flow effectively sweeps through the tank, as shown in Figure 5. The conductive mechanism is minimal. The steady state is attained in a correspondingly shorter time. The steady-state temperatures throughout the tank are close to the temperature of the influx prefixed at the bottom inlet. The thermal boundary layer near the outlet disk becomes thin. The region in which conduction is significant is now confined to a narrow strip near the outlet disk.

The thermal conductance of the vertical sidewall, f, which is incorporated in coefficient B_r in Equation 4, basically determines the fluid transport carried by the vertical sidewall boundary layer.[5,6,8,9] As the thermal forcing at the sidewall decreases, a smaller amount of mass transport is carried through the vertical buoyancy layer. This, in turn, implies that the overall thermal convective activities weaken, as manifested by the global meridional circulation. Figure 6 demonstrates that, as f decreases, the temperature field in the interior is more directly influenced by the overall uplifting motion of the through-flow. The steady-state features in the main body of fluid are akin to a situation in which a vertical uniform flow passes throughout the tank with a perfectly insulated sidewall.

A large conductance at the sidewall gives rise to a strong buoyancy layer transport. Consequently, the overall convective activities are intense. These lead to an effective stratification of the interior fluid (see Figure 7).

The effect of the fluid diffusivity is represented through C_r in Equation 4. Figure 8 shows that, as the relative magnitude of κ increases, the influence of the conductive mechanism becomes more pronounced. Slow temperature changes are noticeable during the latter phase toward the steady state, governed mainly by the diffusive processes. The tendency toward a conduction-controlled, linear temperature profile is discernible. The temperature field in the main interior region varies smoothly,

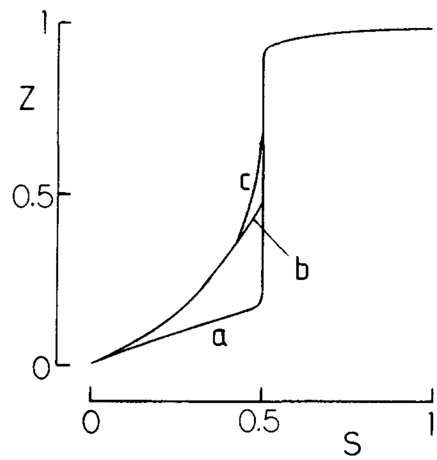

Figure 7 Same as in Figure 3, except $A_r=1.0$, $B_r=5.0$, $C_r=0.02$. Times t are (a) 0.10; (b) 0.40; (c) 0.94

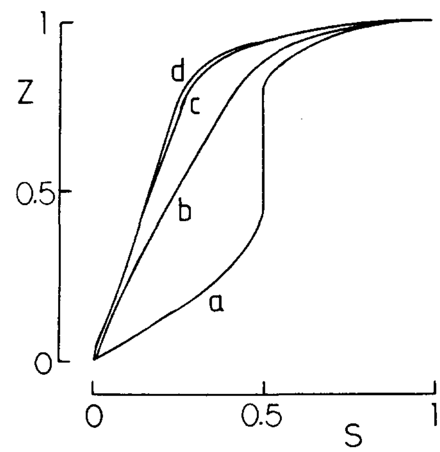

Figure 8 Same as in Figure 3, except $A_r=1.0$, $B_r=1.0$, $C_r=0.1$. Times t are (a) 0.10; (b) 0.50; (c) 1.10; (d) 2.36

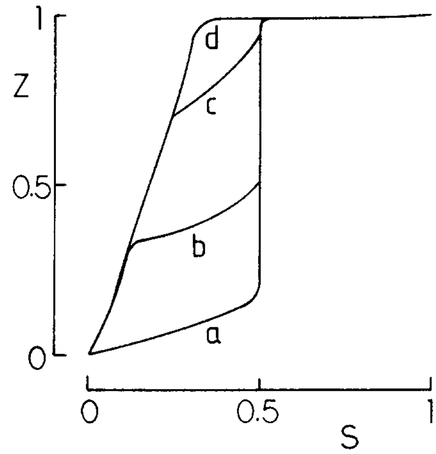

Figure 9 Same as in Figure 3, except $A_r=1.0$, $B_r=1.0$, $C_r=0.002$. Times t are (a) 0.10; (b) 0.40; (c) 0.80; (d) 1.35

and the thickness of the thermal boundary layer at the outlet disk is appreciable. In fact, under a large value of κ, the thermal boundary layer itself becomes less distinct.

When the fluid diffusivity is very small, the temperature field is determined primarily by the convective influences (see Figure 9). The temperature front is clearly seen. The steady state is characterized by a linearly stratified interior and a very thin thermal boundary layer near the outlet disk.

Conclusion

The effects of various external parameters on the development of stratification inside the tank have been examined. As the through-flow increases, the convective activities become an important factor in the determination of the behavior of the contained fluid. The sidewall conductance controls the mass transport carried by the vertical boundary layer. As the fluid diffusivity increases, the influence of the conductive mechanism becomes pronounced. It is demonstrated that the introduction of a controlled vertical through-flow is indeed an efficient means to attain a stratified fluid system.

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